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Multifrequency Coordination in Bimanual Tapping: Asymmetrical Coupling and Signs of Supercriticality

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The circle map provides a general mathematical model for the mode-locking behavior observed in systems of coupled oscillators. From this theoretical perspective, multifrequency tapping was studied. Three experiments were conducted in which skilled drummers participated. The results were in qualitative agreement with the dynamical features of the circle map. The stability of behavior was affected by the movement frequency at which the multifrequency relations were performed. Attraction to lower order ratios (predominantly showing Farey relations) was observed. In some situations bistability and hysteresis occurred, implying that the system was situated in the supercritical domain of the circle map where resonance zones overlap. Furthermore, the results suggest that multifrequency tapping is characterized by an asymmetrical coupling in that the influence of the fast hand on the slow hand is the strongest.

Tasks in which two limbs move at different frequencies, so-called multifrequency tasks, tend to be very difficult. Tapping polyrhythms is a case in point. Polyrhythms are frequency ratios that cannot be simplified to ratios with one as a numerator or denominator (e.g., 2:3, 3:5, and 4:11). Tapping a 3:5 polyrhythm implies that the hands move at frequencies that are related in such a way that one hand taps three times in the same interval in which the other hand taps five times. The performance of polyrhythms has been investigated in a number of studies (e.g., Deutsch, 1983; Jagacinski, Marshburn, Klapp, & Jones, 1988; Klapp et al., 1985; Summers, Ford, & Todd, 1993; Summers & Kennedy, 1992; Summers, Rosenbaum, Burns, & Ford, 1993). In general, higher order ratios (consisting of larger numerators and denominators) are performed with larger intertap variability than lower order ratios. Furthermore, evidence has been reported for an integrated type of control: The two hands do not move independently of each other but interact. This interaction is incorporated in most of the timekeeper models for the coordination of rhythmic performance (e.g., Jagacinski et al., 1988; Summers, Rosenbaum, et al., 1993; Vorberg & Hambuch, 1984).

In contrast, it has been argued that the temporal order observed in bimanual rhythmic performance emerges from the properties of dynamical oscillators (e.g., Kelso, Holt, Rubin, & Kugler, 1981; Kugler, Kelso, & Turvey, 1980). In a series of studies on rhythmic finger movements, Kelso and his colleagues provided evidence that, in such tasks, the movement system handles its large number of internal degrees of freedom by functionally organizing itself as a nonlinear system of coupled oscillators. If the movement frequency of two oscillating index fingers moving in antiphase (simultaneous contraction of nonhomologous muscle groups) was gradually increased, an abrupt switch to the in-phase relation (simultaneous contraction of homologous muscle groups) between these limbs was observed at a critical frequency. If the fingers started in the in-phase coordination, no such transition was observed (e.g., Kelso, 1984; Kelso & Scholz, 1985; Kelso, Scholz, & Schöner, 1986; Kelso & Schöner, 1988). This phenomenon was captured by a model of two nonlinearly coupled nonlinear oscillators (referred to here as the Haken–Kelso–Bunz [HKB] model) that describes the global behavior of the system in terms of a single *collective variable*: relative phase (Haken, Kelso, & Bunz, 1985). The value of this collective variable depends on the initial conditions and the *control parameter,* movement frequency. In the HKB

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model, this parameter is assumed to affect the ratio between the two coupling parameters. The overall coupling strength was indeed demonstrated to be inversely related to movement frequency (Schmidt, Shaw, & Turvey, 1993; Sternad, Turvey, & Schmidt, 1992).

In the HKB model, the in-phase and antiphase modes of coordination are differentially stable: In-phase is more stable than antiphase. In addition, the weakening of the coupling between the limbs (resulting from an increasing movement frequency) leads to a decline of the stability of the two phase relations. Eventually the antiphase pattern no longer provides sufficient stability, and the system is attracted to the remaining stable pattern: the in-phase relation. The model proved to be generalizable to different systems of oscillators, including arm–leg (Kelso & Jeka, 1992), arm–visual stimulus (Byblow, Chua, & Goodman, 1995; Wimmers, Beek, & van Wieringen, 1992), and two legs belonging to different people (Schmidt, Carello, & Turvey, 1990). These results suggest that the coordination dynamics between oscillators is abstract and necessarily informational in nature (Kelso, 1994).

Evidence has been provided that similar styles of coordination as observed in 1:1 frequency tasks underlie multifrequency behavior in human movement (Beek, Peper, & van Wieringen, 1992; Kelso & deGuzman, 1988; Peper, Beek, & van Wieringen, 1991, in press; Treffner & Turvey, 1993). The behavior observed when the limbs are required to move at different frequencies shares many interesting properties with the behavior obtained in nonlinear systems of coupled oscillators. In the present article, three experiments are discussed in which multifrequency tapping performance was examined from the perspective of the physical–mathematical theory of coupled (nonlinear) oscillators. To explain the theoretical relevance of the experimental manipulations and observations, the next section introduces the *circle map,* a general discrete model adopted from nonlinear dynamics that provides insight into the differential stability of multifrequency behavior as well as the kind of transitions that are to be expected if stability is lost. Subsequently, this general model is related to multifrequency behavior as observed in human movement.

Circle Map Dynamics and the Farey Mode-Locking Hierarchy

Oscillator theory suggests that when two oscillators with different eigenfrequencies (i.e., natural frequencies) are coupled, their interactions may result in attraction to a certain frequency ratio, depending on the ratio between the eigenfrequencies and the strength of the coupling. A system of coupled oscillators may be considered a generalized version of the more familiar instance of a periodically forced oscillator (Epstein, 1990). Hence, the general analysis of coupled oscillators may be simplified by the use of circle maps, which represent the behavior in a relatively simple fashion. A one-dimensional circle map is a first-order difference equation in which the system's behavior is studied in discrete steps. Its general form reads as follows:

$$X_{t+1} = F(X_t). \tag{1}$$

Thus, the value of variable X at time $t + 1$ is a function of its value at time t; at $t + 2$, it is a function of its value at $t + 1$, and so forth. In other words, the variable is mapped onto itself or "iterated." Circle maps describe the influence of a periodic external force on the phase of an oscillator. In general, circle maps are defined through

$$\Theta_{n+1} = f_{\Omega}(\Theta_n) = \Theta_n + \Omega + g(\Theta_n), \tag{2}$$

where

$$g(\Theta_n) = g(\Theta_n + 1)\ (\text{modulo } 1) \tag{3}$$

(using the convention of a period of 1 rather than 2π; $0 \leq \Theta < 1$). The variable Θ_n represents the phase of the oscillating system measured stroboscopically (at strobe n) at periodic time intervals $t_n = 2\pi n/\omega$, with the frequency of the external force (ω) as a clock. As in Equation 1, the state of the system is mapped onto itself. The ratio between the eigenperiod of the external forcing and that of the forced oscillator (T_e/T_o) is represented by Ω, referred to as the *bare winding number.* The periodic forcing influences the iteration of Θ through the coupling function g (Equation 3). The effect of the forcing is a function of the phase of the forced system (Θ_n). Because the phase shift $\Theta_n \rightarrow \Theta_n + 1$ represents a full rotation, it functions as the periodic property of the coupling function. Here we consider the smoothest periodic map: the sine circle map, in which the coupling function $g(\Theta_n)$ is defined as $(K/2\pi)\sin 2\pi\Theta_n$. The iteration of the map is conveniently described by its *dressed winding number:* $W(K, \Omega) = (\Theta_n - \Theta_o)/n$ in the limit as the number of iterations n approaches infinity. In other words, W represents the mean number of rotations per iteration (i.e., the ratio between the forcing period and the resulting period of the forced oscillator). Under iteration, the variable Θ_n may converge to a series that is periodic, with W being rational; quasiperiodic, with W being irrational; or chaotic, with the series behaving irregularly. The quantity W is determined by the amplitude of the coupling function K (the coupling strength) and Ω. Rational Ws (indicating resonance or frequency locking) are obtained for ranges of Ω. These ranges or resonance regimes (*Arnold tongues*) are presented for continuously varying K in a *regime diagram* (Figure 1): The coupled oscillators are frequency locked in regimes, the widths of which vary as a function of the strength of the interaction between the oscillators (see Jackson, 1989; Jensen, Bak, & Bohr, 1984).

Depending on the initial conditions of K and Ω, the circle map demonstrates a variety of behaviors (periodic, quasiperiodic, and chaotic). In relation to the present purposes, it is important to classify two major domains in the regime diagram, each of which has its own characteristics. Under the line $K = 1$, the sine circle map behaves monotonically, and none of the periodic tongues overlap: The regions between the periodic tongues correspond to quasiperiodic behavior. The system is attracted toward a certain ratio if its initial conditions fall within the boundaries of the tongue (the *basin of attraction*) corresponding to a particular value

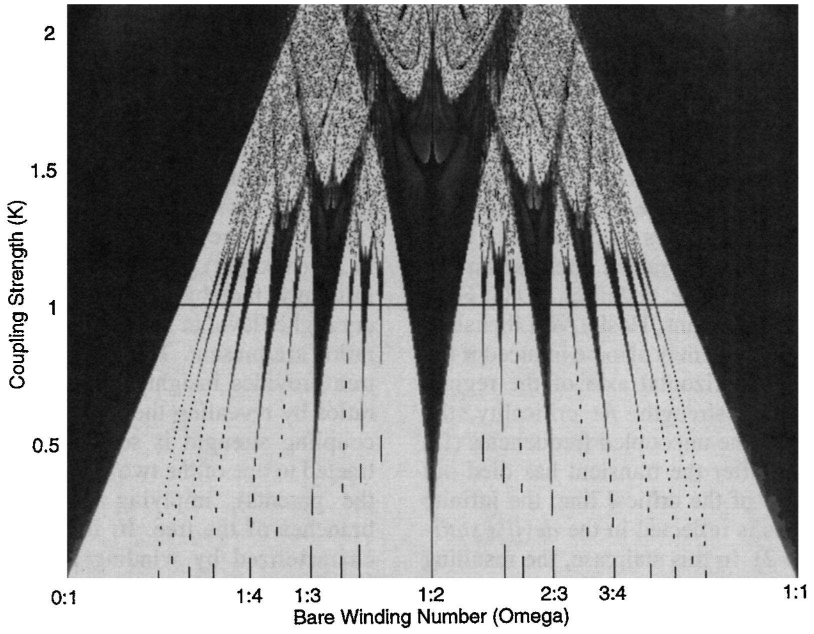

Figure 1. Regime diagram obtained for the sine circle map. For a particular coupling strength (K), specific mode locks occur within ranges of Ω (the ratio T_e/T_o when the oscillators are uncoupled [the bare winding number]). Arnold tongues for ratios up to Farey Level 8 are displayed. Darker regions correspond to more stable behavior.

of K. Consequently, wide Arnold tongues are more stable than narrow ones. In general, lower order frequency ratios (consisting of smaller numerators and denominators) are more stable than higher order ones. It is important to realize that the diagram is filled with infinitely many tongues belonging to all possible rational ratios, most of them extremely narrow and not providing much stability. As K increases, the tongues grow wider, annexing more and more of the space left for irrational ratios. At $K = 1$, all irrational space is incorporated in the basins of attraction of the rational ratios. From here onward periodic tongues start to overlap, and no quasiperiodic behavior can occur. Further strengthening of the coupling may result in chaotic behavior (erratic jumping between overlapping regimes; see Bak, Bohr, & Jensen, 1985). The onset of overlap between Arnold tongues is referred to as *criticality.* Above criticality, apart from periodic behavior, bistable solutions,[1] hysteresis, and chaotic behavior may be observed as a result of the overlap of the Arnold tongues (Glazier & Libchaber, 1988; Haucke & Ecke, 1987; Hilborn, 1994).

Because noise is always present in real systems, Arnold tongues may not be wide enough to resist the fluctuations in behavior. As a consequence, noise-induced transitions to nearby stable (in general lower order) ratios may be observed. In the supercritical domain, transitions to other ratios may also be the result of the overlap between tongues. (In these transitions, too, noise plays an important role.) To evaluate whether the system is indeed attracted to a nearby lower order ratio, it is important to know the spatial layout of the Arnold tongues in the regime diagram. Organizing the subcritical ($K < 1$ for the sine circle map) mode lockings reduces to organizing rational numbers on the unit interval. This can easily be accomplished by using the number-theoretical concept of the *Farey sequence.* In a Farey sequence of order n (referred to as F_n), all rational ratios with a denominator smaller than $n + 1$ are ordered within unity. Between every two ratios P_1:Q_1 and P_2:Q_2 adjacent in F_n (referred to as the *parent* ratios), infinitely many higher order ratios are situated. The lowest order of these intermediate ratios is $(P_1 + P_2)$:$(Q_1 + Q_2)$ (Farey's theorem; Hardy & Wright, 1965). This ratio is called the *mediant,* and its order is larger than n. Systematic application of Farey's theorem ensures the correct ordering of rational ratios in Farey sequences of increasing order. The unit interval may be dissected into Farey intervals assigned to rational ratios. In F_n, the interval belonging to a specific rational ratio is confined by the mediants of this ratio and the two adjacent ratios. However irregularly the Farey fractions of order n are

[1] Note that *bistability* here refers to the situations in which, for a specific set of initial conditions (Ω and K), two stable solutions are present. This meaning of the term is different from the bistable solutions present in the HKB model, in which, although both the in-phase and the antiphase patterns provide stability, the occurrence of either behavioral mode is determined by the initial phase relation and the ratio between the coupling parameters. According to the HKB terminology, the regime diagram reveals multistability in the subcritical domain as well.

spaced between 0 and 1, the Farey intervals have a specific measure of uniformity. This Farey dissection reveals that the widths of the intervals (determined by the distance between the two mediants associated with each ratio) decrease as the order of the ratios increases (Ayoub, 1963). In a similar vein, lower order Arnold tongues are wider than higher order ones and, thus, are more stable.

If coupling strength is manipulated, transitions from less stable ratios to more stable ones can be induced. In several physical systems, the transition routes to chaos have been investigated that resulted from a gradual increase in the strength of coupling (e.g., Bohr, Bak, & Jensen, 1984; Fein, Heutmaker, & Gollub, 1985; Stavans, Heslot, & Libchaber, 1985). Transitions between ratios may also be induced if the system is guided along the horizontal axis of the regime diagram at a fixed coupling strength. At criticality, the relation between the ratio of the uncoupled frequencies (Ω) and the behavior observed after the transient has died out reflects the fractal structure of the critical line; the infinite number of periodic regimes is reflected in the *devil's staircase* structure (see Figure 2). In this staircase, the resulting ratios (*Ws*) form plateaus as a function of Ω, reflecting the regions of attraction for each ratio. Below criticality, the staircase is incomplete as a result of the regions of quasiperiodic behavior (Bak, 1986). The devil's staircase structure has been identified in numerical simulations of a variety of periodically forced systems (e.g., electrical circuits: Bohr et al., 1984; excitable chemical systems: Dolník, Marek, & Epstein, 1992; and neurons: Harmon, 1961). In addition, empirical examinations of such systems have revealed similar ratio plateaus induced by scaling the ratio of the eigenfrequencies Ω (e.g., Belykh, Pederson, & Soerensen, 1977; Finkeová, Dolník, Hruda, & Marek, 1990; Maselko & Swinney, 1985).

The mathematical structure of the *Farey tree* can be constructed by systematically applying Farey's theorem (Figure 3); Cvitanović, Shraiman, & Söderberg, 1985; González & Piro, 1985). Following González and Piro, the ratio 1:2 (the mediant of the ratios 0:1 and 1:1) is situated at the zeroth level in the tree. The ratios at the next level in the tree are obtained by application of the Farey sum (i.e., summation of the numerators and denominators, respectively): Farey summation of 0:1 and 1:1 (parents) results in 1:2 (mediant); Farey summation of 1:2 and 1:1 (parents) results in 2:3 (mediant), and so forth. This process can be continued infinitely, thereby producing new intermediate ratios at every higher level in the tree. At every *k*th level in the tree, $2^{|k|}$ ratios are present. The mathematical structure of the Farey tree provides insight into the organization of the rational ratios by revealing the transition routes that are expected if coupling strength is scaled. In general, the system is attracted to one of the two nearest lower order regimes (one of the parents), implying that the transitions follow the branches of the tree. If, for instance, the behavior can be characterized by winding number 3:5, the nearest lower order ratios (parents) are 1:2 and 2:3 (Farey's theorem). A transition to one of these two closest lower order ratios is referred to as a *unimodular* (or *mod1;* see Treffner & Turvey, 1993) transition, based on the fact that $|P_1Q_2 - P_2Q_1| = 1$ (Allen, 1983; Hardy & Wright, 1965).

As mentioned earlier, the one-dimensional circle map provides a general picture of the behavior of a periodically forced oscillator that carries over to situations in which two oscillators are influencing each other (bidirectional coupling). Indeed, in physical and chemical systems of bidirectionally coupled oscillators, behavior similar to that of the circle map has been observed (e.g., Crowley & Field, 1986; Haucke & Ecke, 1987; Marek & Stuchl, 1975; Nakajima &

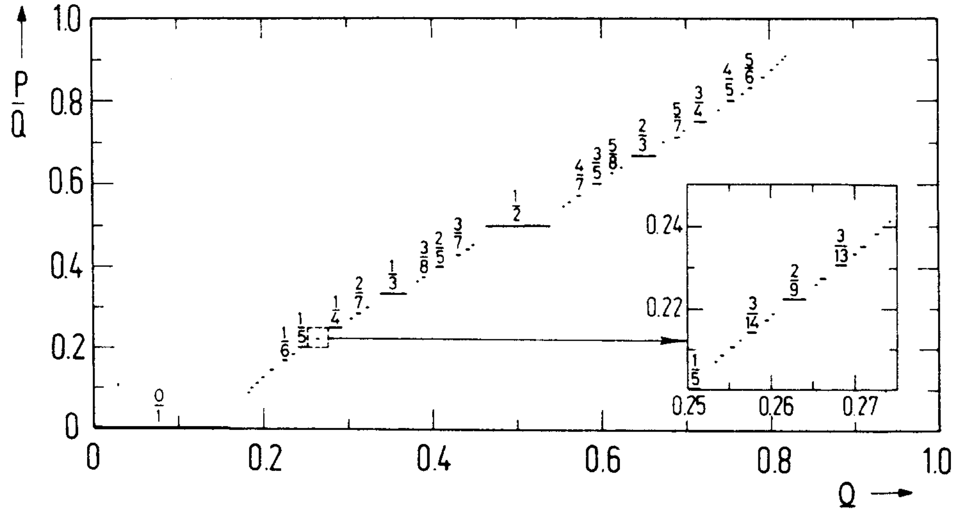

Figure 2. Devil's staircase: the mode-locking structure of the sine circle map at $K = 1$. The dressed winding numbers $P{:}Q$ are presented as a function of Ω. The self-similar (fractal) structure is exemplified by the inlay figure. From "Transition to Chaos by Interaction of Resonances in Dissipative Systems: I. Circle Maps," by M. H. Jensen, P. Bak, and T. Bohr, 1984, *Physical Review A, 30,* p. 1962. Copyright 1984 by The American Physical Society. Adapted with permission.

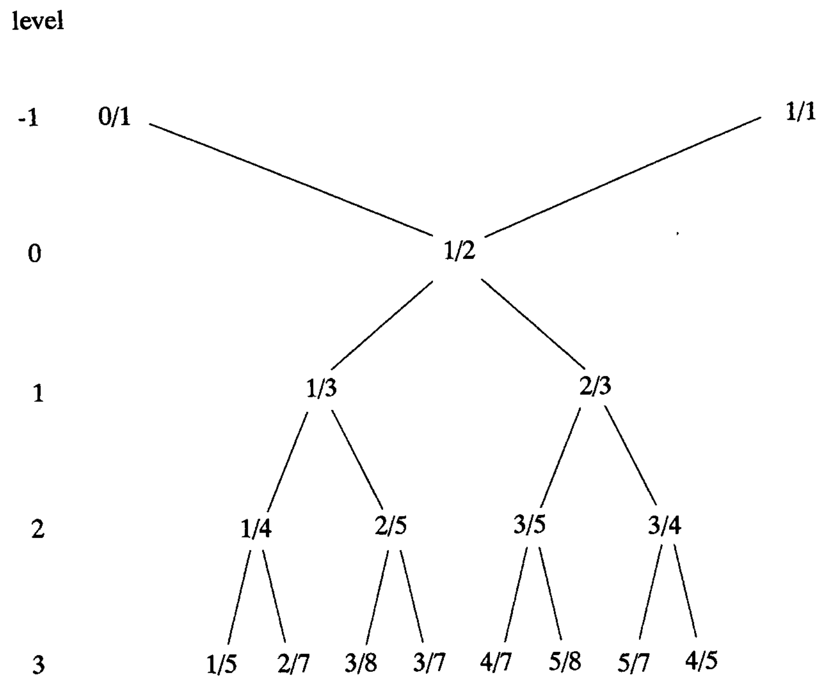

Figure 3. First five levels of the Farey tree.

Sawada, 1980). Hence, like others (deGuzman & Kelso, 1991; Kelso, deGuzman, & Holroyd, 1990; Treffner & Turvey, 1993), we deem the circle map an appropriate tool in the qualitative study of systems in which the coupled oscillators force each other, as may be the case in bimanual rhythmic tapping.

Dynamics of Multifrequency Behavior in Human Movement

Oscillator theory thus describes the differential stability of frequency ratios. The resonance regions are a function of the strength of the interaction between the oscillators. Multifrequency mode locking is observed not only in physical and chemical systems but also in the behavior of biological systems, including cardiac rhythms (e.g., Guevara & Glass, 1982; Moe, Jalife, Mueller, & Moe, 1977), circadian control of ovulation (see Winfree, 1990), interactions between pacemaker neurons (Perkel, Schulman, Bullock, Moore, & Segundo, 1964), and the coordination of breathing and locomotion in running (Bramble & Carrier, 1983) and cascade juggling (Beek, 1989a; Beek & Turvey, 1992). In addition, a number of studies have provided evidence that, in bimanual production of multifrequency relations, the resonance between the coupled oscillators constrains the behavioral patterns. Kelso and deGuzman (1988) found marked differences in variability between ratios: Lower order ratios were performed with less variability than higher order ratios and, moreover, attracted the system when the required higher order ratios could not be performed. The latter result indicated that the higher order ratios did not provide sufficient stability, resulting in the attraction to a more stable pattern. Treffner and Turvey (1993) demonstrated that such transitions to lower order ratios generally followed the unimodular relations in the Farey tree. Peper et al. (1991, in press; see also Beek et al., 1992) drove the system through the regime diagram by gradually increasing movement frequency (related to decreasing coupling strength; Sternad et al., 1992). This manipulation resulted in bifurcations from one ratio to another; the resulting weakening of the coupling induced loss of stability of the required ratio, and the system was attracted to a nearby ratio that was still sufficiently stable. In general, these transitions also followed the branches of the Farey tree. Systematic deviations from these predicted transition routes were interpreted as being the result of changes in the intrinsic dynamics, which were due to learning (see Zanone & Kelso, 1992).

On the basis of the common observation that, in multifrequency performance, the hands show tendencies toward momentary phase attraction (in the antiphase or the in-phase mode; e.g., Byblow & Goodman, 1994; deGuzman & Kelso, 1991; Summers & Kennedy, 1992; Summers, Rosenbaum, et al., 1993), Kelso et al. (1990; see also deGuzman & Kelso, 1991) extended the circle map by introducing an additional parameter, resulting in the *phase attractive circle map.* This third parameter allows for momentary phase attraction but also has an effect on the widths of the tongues and the critical value of the coupling parameter, indicating that the behavioral patterns arise from the competition between extrinsic (coupling to a forcing oscillator) and intrinsic (phase attraction) dynamics (deGuzman & Kelso, 1991).

In the preceding studies, the behavior of the system was never completely frequency locked; some variability in performance was typically present. This variation is important, because the system is less flexible if strictly mode locked. Previous studies (Beek, 1989b; Kelso et al., 1990) have suggested that the system is situated close to the border of mode-locking regions, exhibiting the more flexible form of "relative coordination" (Von Holst, 1937/1974). In this way, the system can function in a stable fashion while still being able to adapt itself to changing circumstances.

The fact that the system is attracted to lower order ratios and, especially, the fact that sudden transitions are observed when movement frequency is scaled strongly support the assumption that rhythmically coordinated limbs act as a system of coupled oscillators. However, application of the theory of coupled oscillators to multifrequency behavior in humans has to be pursued with some caution. Some clear differences exist between physical (and chemical) oscillators and the human movement system. An important factor is, of course, the influence of learning and intention in the human system. Kelso and his coworkers introduced the term *behavioral information* to account for their effects. To-be-learned and intended behavioral modes were modeled as attractor states by means of the same collective variables that were used to characterize the observed coordination patterns (Schöner & Kelso, 1988a, 1988b; Schöner, Zanone, & Kelso, 1992). Their empirical results justified this modeling step (see Scholz & Kelso, 1990; Zanone & Kelso, 1992).

Another important difference is the fact that human limbs do not have fixed eigenfrequencies, because stiffness is an adjustable parameter. Physical and chemical oscillators attain specific eigenfrequencies when uncoupled, but this is not necessarily true for oscillating limbs. It is possible that, in the experiments presented by Kelso and deGuzman (1988) and Treffner and Turvey (1993, Experiment 1), decoupling would not have resulted in a return to the preset frequency. Because the circle map is based on the interaction between eigenfrequencies of oscillators, this is a crucial difference. Application of circle map dynamics in human movement may be facilitated if the analogy between coupled physical oscillators and rhythmically coordinated limbs is amplified. One option is to increase the inertia of the system, thereby decreasing the range of possible movement frequencies (see Treffner & Turvey, 1993, Experiments 2 and 3). Another way is to specify the required rhythms throughout the trial and to instruct the participant to synchronize his or her movements to the stimuli. This procedure results in an intentional analogue of the eigenfrequencies. In the experiments reported here, we used the latter option.

Transitions in Bimanual Multifrequency Behavior

The results of Kelso and deGuzman (1988), Peper et al. (1991, in press), and Treffner and Turvey (1993) demonstrated that the two hands interact and that the behavior is attracted to a nearby ratio (predominantly showing unimodular relations) if the required ratio is not sufficiently stable. In interpreting these findings, it is important to realize that the system has both a deterministic structure (i.e., the structure described by the circle map) and stochastic properties. Treffner and Turvey suggested that the transitions observed resulted from marginally small tongue widths of the required ratios. If the intended ratio offers little stability (i.e., the Arnold tongue is very narrow), stochastic fluctuations in performance may move the system out of the required tongue. If, subsequently, the system ends up in a tongue that provides sufficient stability to resist the fluctuations, it remains there and behaves accordingly. As explained earlier, however, the circle map reveals that the transitions may also result from overlap between tongues. Such transitions may be observed only in the supercritical domain, in which Arnold tongues overlap. If the resonance zone of the to-be-performed ratio is overlapped by a lower order tongue (bistability), the system may (a) remain in the intended resonance zone, (b) be attracted to the overlapping lower order ratio permanently, or (c) alternate aperiodically between the two overlapping tongues. Which of these three possibilities occurs is a function of the initial values of K and Ω and the stochastic properties of the system.

Because the widths of the regions of attraction are a function of both the ratio of the eigenfrequencies (x-axis in Figure 1) and the strength of the coupling between the hands (y-axis in Figure 1), there are, in principle, two ways to induce transitions in the system. One is to manipulate the coupling strength by manipulating movement frequency (inversely related to coupling strength; Sternad et al., 1992), thus driving the system through the regime diagram along the vertical axis (Peper et al., 1991, in press). The other way is to manipulate the ratio between the eigenfrequencies of the oscillators to guide the system along the horizontal axis. Thus, structures resembling the devil's staircase may be obtained, allowing insight into the widths of the attraction regions corresponding to rational frequency ratios. If these two manipulations are combined, that is, guiding the system along the horizontal axis (by manipulating the to-be-performed ratios) at different movement frequencies (related to different coupling strengths on the vertical axis), the effect of movement frequency on the widths of the periodic regimes can be examined. The results may be informative about the dynamics of bimanual rhythmic performance and the nature of the transitions observed therein.

Experiment 1

To strengthen the analogy with the circle map (i.e., to make the coupling asymmetrical), we assigned one hand the role of "forcing oscillator" with the aim of guiding the participants along the horizontal axis in the regime diagram while keeping the value of the coupling parameter fixed. The performance of the forcing hand was facilitated by means of the experimental conditions, which were chosen so as to support optimal synchronization with this hand. The preferred (right) hand was selected for this purpose (e.g., see Byblow et al., 1995; Peters, 1980, 1985), and the participant was instructed to pay close attention to that hand while synchronizing the taps to an auditory stimulus (which is more salient than a visual stimulus in synchronization tasks; see Barlett & Barlett, 1959; Dunlap, 1910; Fraisse, 1948). The forcing hand performed at a number of frequency levels that were assumed to be related to different levels of coupling strength. The frequency of the other hand (the "forced oscillator") was slowly increased or decreased. To minimize the integration of the two frequencies in a coherent rhythm (see Jagacinski et al., 1988), we chose a different stimulus modality for this hand: a flashing light. Given the general model assumptions (the unit oscillators are not affected by differences in movement frequency, whereas the strength of coupling between them is inversely related to the movement frequency at which the frequency ratio is performed), the proposed method of scaling the intended ratios along the horizontal axis of the regime diagram allows for examination of the widths of the resonance zones at fixed coupling strengths.

Method

Participants

Three skilled male drummers (28, 29, and 33 years of age) participated. On the basis of the way in which they were used to drumming, they were right-handed (playing the hi-hat using the right hand and left foot, the snare using the left hand, and the base using the right foot). They were paid for their services.

Experimental Setup

The hand movements were measured with a two-dimensional Selspot system that recorded the position of two light-emitting diodes (LEDs) positioned on the tips of the middle fingers (sample frequency = 312 Hz). A microcomputer (IBM PS/2 40 SX) controlled the Selspot system and a stimulus device that could produce both auditory and visual stimuli. The accuracy of stimulus onset was ±1.5 ms. The taps were performed on a low-resonance marble tabletop surface.

Two stimulus trains were presented, one through the right channel of a headphone (Sennheiser HD 520 II) and one by means of a green LED (diameter of approximately 1 cm) placed near the tapping position of the left hand. Each stimulus lasted 50 ms. The interval between the onsets of beeps was fixed during a trial. Eight frequency levels were used in the experiment; the periods between the onsets of the beeps (hereafter referred to as *forcing periods*) ranged from 300 to 1,000 ms in 100-ms steps. For the visual stimulus train, the interstimulus periods gradually increased or decreased in steps of 50 ms. Each frequency was presented for approximately 10 s so that frequency platforms were formed. The interstimulus period ranged from 300 to 1,400 ms. The range was divided into three separate parts to limit the duration of the trials. The resulting visual stimulus patterns were categorized as high (300–700 ms, 20 stimuli per platform, mean platform duration of 10 s, and trial duration of 90 s), medium (700–1,100 ms, 11 stimuli per platform, mean platform duration of 9.9 s, and trial duration of 89 s), and low (1,100–1,400 ms, 10 stimuli per platform, mean platform duration of 12.5 s, and trial duration of 87.5 s). These trains could be presented both back and forth (decreasing period and increasing period). In combination with the auditory stimulus, the range of tested ratios was different for the eight auditory conditions. The limits of these ranges were situated between 3:14 and 1:1 (forcing period = 300 ms) and between 5:7 and 10:3 (forcing period = 1,000 ms). The resulting 184 ratio–frequency combinations are presented in Table 1.

Procedure

The participant was instructed to tap with his hands on the marble tabletop (rotation about the wrist) while the lower arms were resting on its surface. The right (preferred) hand had to synchronize to the auditory stimulus. The participant was told that the frequency of this stimulus train would not change during the trial. He was instructed to pay close attention to the right hand (forcing oscillator) and to tap with that hand at the prescribed frequency, even if this would hamper the performance of the other hand. During an experimental session, the participant was reminded of this repeatedly. The left hand (forced oscillator) had to synchronize to the visual stimulus. The participant was informed about the range of frequencies before each trial (high, medium, or low and speeding up or slowing down).

The experiment was run in two sessions. During a session, the frequency of the visual stimulus either increased or decreased (the order in which these two conditions were presented was semicounterbalanced across participants). The trials were blocked for "forcing frequency" conditions to facilitate performance with the right hand as much as possible. Each forcing frequency block was preceded by a trial presenting the auditory stimulus for the right hand in isolation to accustom the participant to the new frequency. Within such a frequency block, three subblocks of trials were conducted; three trials belonging to a visual frequency range (high, medium, or low) were grouped together. These subblocks were presented in random order within the forcing frequency block.

Before the experimental trials of a session were started, the participant was acquainted with the task by means of practice trials. For two forcing frequencies, the participant received two subblocks of two trials (preceded by the accompanying unimanual practice trial). These practice trials were pseudorandomly selected; the experimenter made sure that the two forcing frequencies differed by at least 300 ms and that each visual frequency range was practiced at least once.

The participant was allowed to take short rests between trials if desired; longer breaks (approximately 15 min) were allowed after two or three frequency blocks (depending on the participant's preferences). Each session lasted about 3 hr.

Analysis

The Selspot data were transformed into Cartesian coordinates (by means of direct linear transformation; see Miller, Shapiro, & McLaughlin, 1980; Shapiro, 1978) and filtered with a second-order recursive Butterworth filter (applied back and forth; cutoff frequency = 20 Hz). We determined the moments of tapping using an algorithm based on the zero crossings in the velocity signal constrained by position and downward peak velocity requirements.

For each frequency ratio platform,[2] we determined the mean intertap intervals, the corresponding coefficients of variation (CVs), and the relative absolute errors of the mean intertap inter-

[2] Because the medium range overlapped with the other ranges (see specification of stimulus patterns), one of the overlapping platforms was selected. The selection was such that the platform preceded by another one was chosen (in other words, the selected platform was never the first one of a range). Depending on the condition (speeding up or slowing down), the data to be used in the analyses were thus selected.

Table 1
Frequency Ratios Presented in Experiment 1

Forced period (ms)	Forcing period (ms)							
	300	400	500	600	700	800	900	1,000
300	1:1	4:3	5:3	2:1	7:3	8:3	3:1	10:3
350	6:7	8:7	10:7	12:7	2:1	16:7	18:7	20:7
400	3:4	1:1	5:4	3:2	7:4	2:1	9:4	5:2
450	2:3	8:9	10:9	4:3	14:9	16:9	2:1	20:9
500	3:5	4:5	1:1	6:5	7:5	8:5	9:5	2:1
550	6:11	8:11	10:11	12:11	14:11	16:11	18:11	20:11
600	1:2	2:3	5:6	1:1	7:6	4:3	3:2	5:3
650	6:13	8:13	10:13	12:13	14:13	16:13	18:13	20:13
700	3:7	4:7	5:7	6:7	1:1	8:7	9:7	10:7
750	6:15	8:15	2:3	4:5	14:15	16:15	18:15	4:3
800	3:8	1:2	5:8	3:4	7:8	1:1	9:8	5:4
850	6:17	8:17	10:17	12:17	14:17	16:17	18:17	20:17
900	1:3	4:9	5:9	2:3	7:9	8:9	1:1	10:9
950	6:19	8:19	10:19	12:19	14:19	16:19	18:19	20:19
1,000	3:10	2:5	1:2	3:5	7:10	4:5	9:10	1:1
1,050	2:7	8:21	10:21	4:7	2:3	16:21	6:7	20:21
1,100	3:11	4:11	5:11	6:11	7:11	8:11	9:11	10:11
1,150	6:23	8:23	10:23	12:23	14:23	16:23	18:23	20:23
1,200	1:4	1:3	5:12	1:2	7:12	2:3	3:4	5:6
1,250	6:25	8:25	2:5	12:25	14:25	16:25	18:25	4:5
1,300	3:13	4:13	5:13	6:13	7:13	8:13	9:13	10:13
1,350	2:9	8:27	10:27	4:9	14:27	16:27	2:3	20:27
1,400	3:14	2:7	5:14	3:7	1:2	4:7	9:14	5:7

Note. In each trial, one of the forcing periods (between successive beeps) was combined with the gradually scaled forced period (between successive visual stimuli).

vals (RAEs; absolute error scaled to intended period) for each hand after the first part of the platform had been excluded from analysis. Because the durations of the platforms differed over the stimulus frequencies and ranges, the excluded part was determined by eliminating the period marked by the first few beeps of the platform (fast: five beeps; medium: three beeps; and slow: two beeps). Division of the mean period observed for the right hand (forcing period) by that observed for the left hand (forced period) resulted in the mean frequency ratio performed during the platform. The behavior was considered to be stable if the CV averaged over the two hands was smaller than or equal to 15%. (This criterion was selected pragmatically on the basis of preliminary analyses of the data.) The percentage of the 1,104 platforms that was not performed in a stable fashion differed across the participants (Participant A, 7%; Participant B, 13%; and Participant C, 22%).

Results and Discussion

The mean RAEs (averaged over participants and conditions) were 1.5% (SD = 2.5%) for the right hand and 5.5% (SD = 7.0%) for the left hand. The corresponding mean CVs were 8.8% (SD = 3.8%) and 13.4% (SD = 7.0%). For each individual participant, we tested whether the forcing (right) hand indeed performed more accurately than the forced (left) hand (i.e., conformed to the task instruction). Because the tested ratios differed over forcing frequency conditions, performance was influenced by different regions of attraction in each condition. For this reason, we interpreted the results at the ordinal scale by performing Wilcoxon signed-rank tests for matched pairs (right and left hand) on the RAEs and CVs obtained for the individual participants in the eight forcing frequency conditions. For each participant, in each of the eight conditions, the forcing (right) hand performed more accurately than the forced (left) hand: $N = 8$, $T = 0$, $p < .005$ (one-tailed) for both CV and RAE.[3]

There were clear individual differences in the degree to which the participants were able to perform the intended ratios. Participant C showed wider regions of attraction than the other 2 participants. Figure 4 shows the results for this participant obtained in one of the frequency conditions. The observed frequency ratios are presented as a function of the intended frequency ratio (with unstable coordinations excluded). If all ratios had been performed as required, all data points would fall on the thin line in the figure (the line $y = x$, representing perfect performance). Deviations from this line represent attractions toward ratios other than the frequency ratio specified by the signals. The obtained structure resembles the devil's staircase, showing the regions of attraction of the stable ratios. As can be seen in Figure 4, the attracting ratios were predominantly lower order ratios such as 1:1, 2:1, and 3:1. This result is in agreement with the general expectation that lower order ratios provide more stability than higher order ratios.

An interesting observation, and one that held for all participants, is that for intended ratios larger than 1.0 (i.e., situations in which the forcing [right] hand moved more slowly than the forced [left] hand), the behavior often locked into one of just a few lower order Arnold tongues.

[3] Note that these results remained significant with the use of two-tailed tests ($p < .01$).

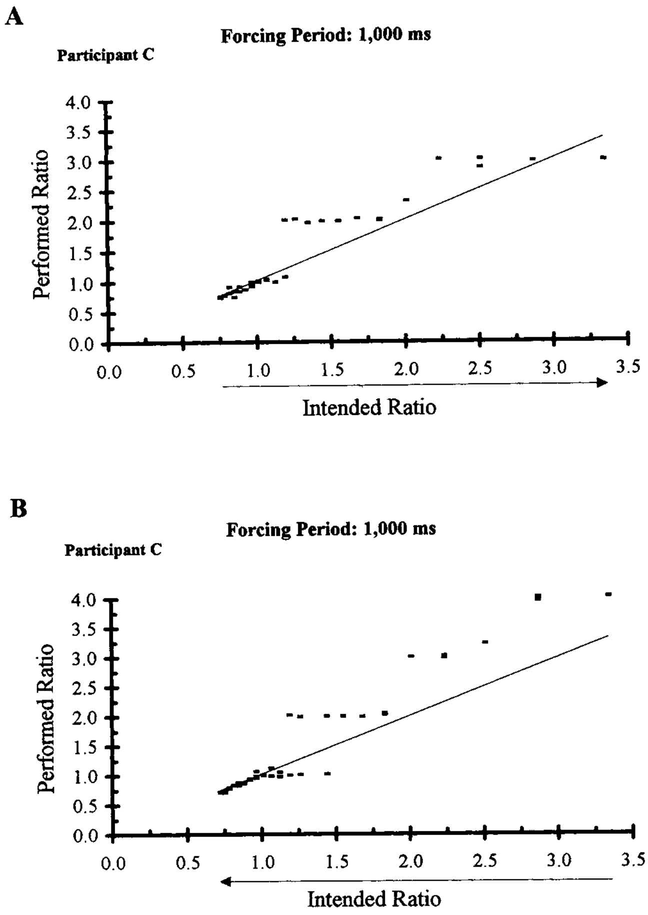

Figure 4. Stable frequency ratios between the forcing hand and the forced hand as a function of intended ratio (obtained for Participant C). The intended frequency of the forced hand either increased (Panel A) or decreased (Panel B). Arrows denote resulting routes along the *x*-axis, and thin lines denote locations of correct responses.

For ratios smaller than 1.0, the participants were largely able to perform higher order ratios (especially for the lower frequency conditions). Participant C showed the largest differences between these two situations.

Figures 4A (speeding up) and 4B (slowing down) represent the two scaling directions. In these conditions, exactly the same required ratios were presented. However, the direction in which the system was guided along the *x*-axis differed (as indicated by the arrows). Comparison of Figures 4A and 4B reveals that attraction to the regions of 1:1 and 2:1 and attraction to the regions of 3:1 and 4:1 overlapped and were history dependent; for the two routes along the *x*-axis, the ranges of intended ratios for which the system showed attraction to certain other ratios were different. This *hysteresis* effect is of utmost importance because, in the one-dimensional circle map, hysteresis is to be expected only if the system is situated above the critical line in the region where Arnold tongues overlap and bistability may occur (Glazier & Libchaber, 1988; Haucke & Ecke, 1987). This interpretation is supported by the large regions of attraction observed for the ratios 2:1 and 3:1 for the low-frequency conditions. The fact that the system is attracted to the 2:1 ratio even if the intended ratio is very close to 1:1 (see Figure 4) seems difficult to explain on the basis of stochastic fluctuations below criticality; according to the latter interpretation, attraction to 1:1 would be much more probable when the intended ratio was located near this zone of resonance. These converging results suggest that the coupling between the hands can be strong enough to situate the system in the complicated supercritical region of the regime diagram.

When movement frequency was increased and, consequently, coupling was weakened, the wide attraction regions demonstrated for the lower order ratios larger than 1.0, as observed for Participant C, decreased in width (see Figure 5). The 2:1 tongue receded, and more attractions to the 1:1

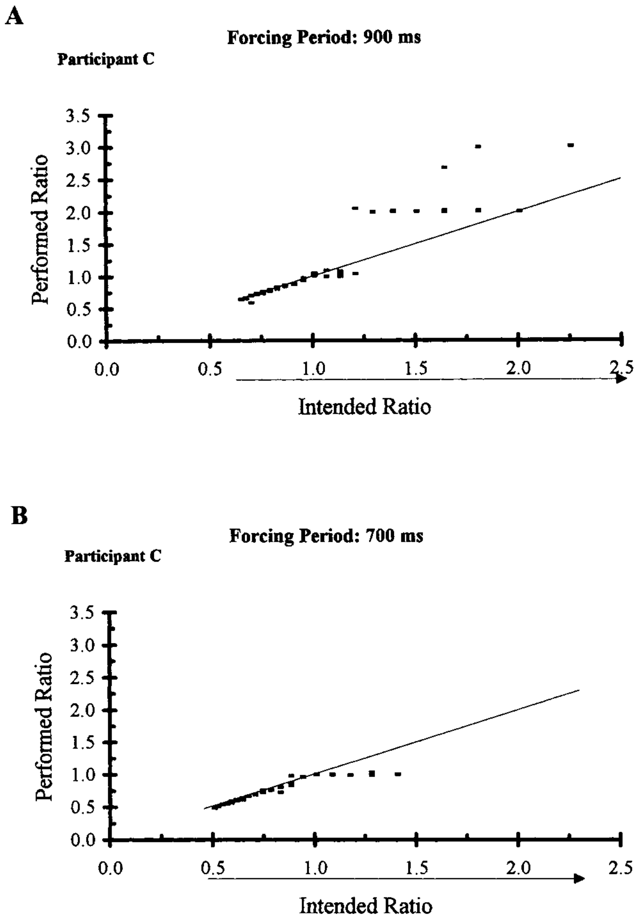

Figure 5. Stable frequency ratios between the forcing hand and the forced hand as a function of intended ratio (obtained for Participant C). The ratios were performed at a slower rate in Panel A than in Panel B (indicated by the forcing periods). The intended frequency of the forced hand increased. Arrows denote resulting routes along the *x*-axis, and thin lines denote locations of correct responses.

ratio were observed. These attractions to 1:1 were probably due to fluctuations driving the system into the nearest tongue. The results suggest that, for Participant C, none of the higher order ratios beyond 1.0 provided any stability at the frequencies tested.

Examination of performance when the forcing hand moved faster than the forced hand (ratios smaller than 1.0) revealed that, for Participant C, more transitions to lower order ratios occurred in the high-frequency conditions than in the lower frequency conditions (cf. Figures 6A and 6B). At the lower frequencies, the participant was able to perform most ratios below 1.0 reasonably well, revealing that the higher order ratios were stable and that their regimes did not overlap (see Figure 6A). As movement frequency was increased and, thus, coupling was weakened, the resonance regimes became narrower, rendering the higher order tongues too small to provide sufficient stability. The system drifted out of these tongues, ending up in the more stable

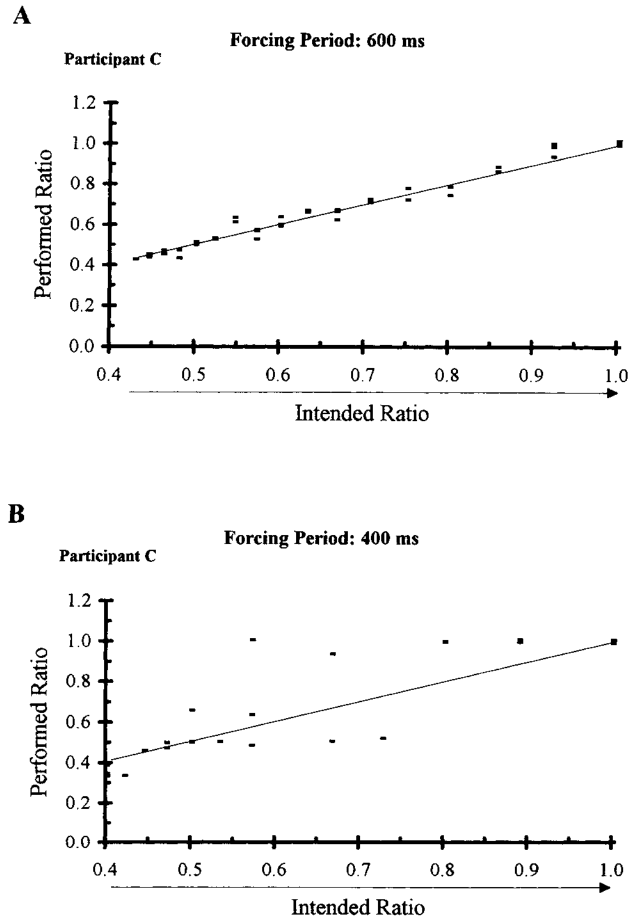

Figure 6. Stable frequency ratios between the forcing hand and the forced hand as a function of intended ratio (obtained for Participant C). The ratios were performed at a slower rate in Panel A than in Panel B (indicated by the forcing periods). The intended frequency of the forced hand increased. Arrows denote resulting routes along the *x*-axis, and thin lines denote locations of correct responses.

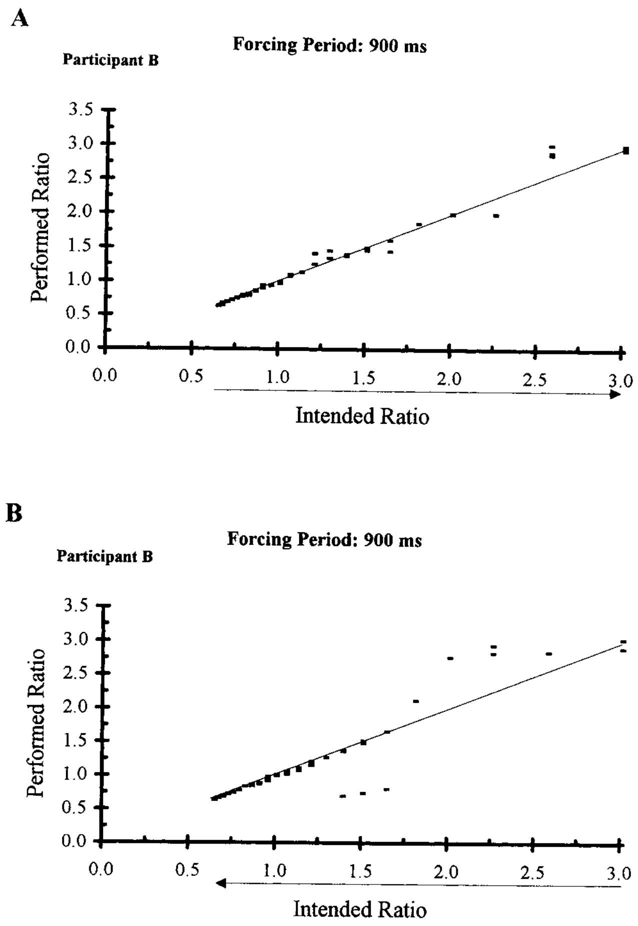

Figure 7. Stable frequency ratios between the forcing hand and the forced hand as a function of intended ratio (obtained for Participant B). The intended frequency of the forced hand either increased (Panel A) or decreased (Panel B). Arrows denote resulting routes along the *x*-axis, and thin lines denote locations of correct responses.

regions for lower order frequency lockings. Figure 6B shows that, for a range of intended ratios, the system was attracted to either 1:2 or 1:1 (and, for a smaller range, to either 1:2 or 2:3), revealing the stochastic nature of the transitions.

The results obtained for Participant C suggest that the system operated above the critical line. In this domain of the regime diagram, only periodic and chaotic behavior can be present (Glazier & Libchaber, 1988). All nonperiodic behavior, thus, is chaotic rather than quasiperiodic: Iterations of Θ lead to an irregular series instead of an irrational winding number. Although the time series that were classified as nonperiodic (by means of our rather crude measure) were fairly irregular (given the large CVs), it would be premature to conclude that they were indeed chaotic. Unfortunately, the time series were too short to determine whether chaotic behavior really did occur. It is possible that, for the coupling strengths tested, the chaotic regions were too narrow to be observed (see Maselko & Swinney, 1985).

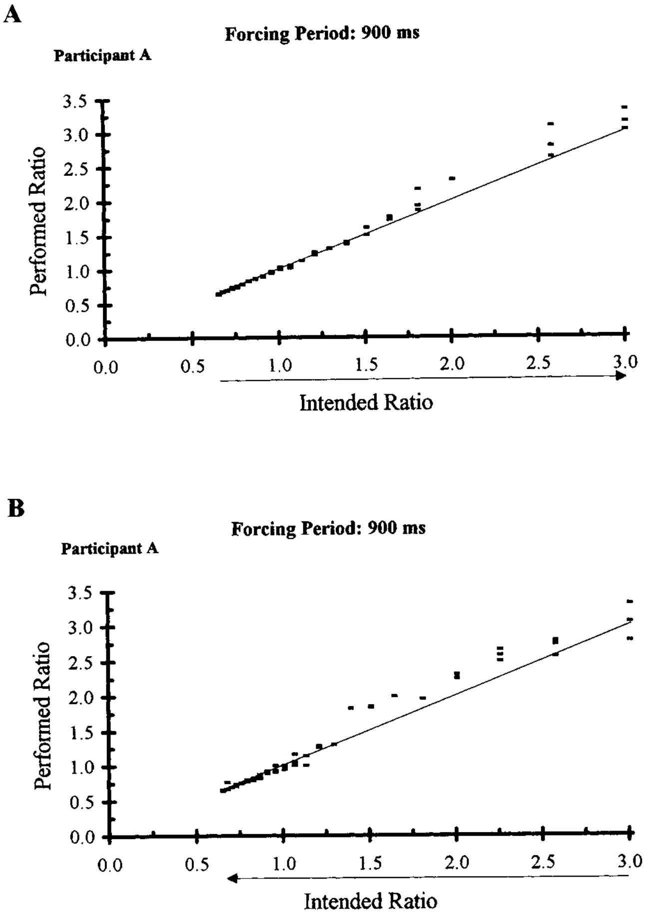

Figure 8. Stable frequency ratios between the forcing hand and the forced hand as a function of intended ratio (obtained for Participant A). The intended frequency of the forced hand either increased (Panel A) or decreased (Panel B). Arrows denote resulting routes along the *x*-axis, and thin lines denote locations of correct responses.

The behavior may turn out to be periodic at a larger time scale, or the system may have jumped between bistable states in a regular fashion.

Although the described effects were by far the most pronounced for Participant C, the results for the 2 other participants were qualitatively similar. For Participant B, 3:1 appeared to be a large region of attraction, indicating overlap of tongues. For this participant, the history dependence of the system was expressed in the hysteresis effect represented in Figure 7. For the higher movement frequencies, the 1:2 and 1:1 tongues attracted the system when the regions for the higher order intended ratios became too small to provide stability. Participant A displayed a high degree of accuracy even in the performance of higher order ratios (see Figure 8). As mentioned, Participant A failed to meet the criterion for stable performance on only 7% of the platforms. As can be seen in the typical staircases for this participant, most ratios were performed correctly. This implies that (a) the coupling was such that the tongues for higher order ratios (such as 9:13) were well developed and (b) the tongues were either not yet overlapping or the overlap was still restricted to ratios of an even higher order (recall that the regime diagram is filled with an infinite number of tongues belonging to all possible rational ratios, including the extremely narrow tongues of ratios of a very high order). The differences between the participants may reflect differences in coupling strength between the hands.[4] In addition, the attractor layout of the intrinsic dynamics of the participants may have been different as a result of experience (see Zanone & Kelso, 1992).

In sum, one may conclude that even (or perhaps especially) among trained drummers, there are vast differences in the ability to perform frequency ratios. All participants showed, albeit in different degrees, attraction to lower order ratios. The attraction to other ratios appeared to be based on both overlap between tongues (for high coupling strengths) and loss of stability of tongues (for low coupling strengths). Whether the system was attracted to a specific ratio depended on movement frequency as well as on the history of the system (hysteresis). These results imply that, at least for Participants B and C, the system was situated above the critical line, where Arnold tongues overlap. With respect to Participant A, it is not possible to decide whether performance involved the subcritical or supercritical domain because no clear bistability or hysteresis effects were observed for this participant.

The data showed, especially for the low-frequency conditions, that if the forcing frequency was lower than the forced frequency (ratios larger than 1.0), the system was more often attracted to a lower order ratio than if the forcing hand moved the fastest (ratios smaller than 1.0). Although it is possible that this difference was due to the experimental conditions, similar behavior has been observed in numerical experiments involving the Brusselator model (M. Dolník, personal communication, November 18, 1993, and September 12, 1994; Kai & Tomita, 1979; Schreiber, Dolník, Choc, & Marek, 1988), which may suggest that this observation is more generic. The Brusselator is a theoretical model of a chemical system that is known to exhibit limit cycle behavior (i.e., chemical oscillator or a "chemical clock"). In line with our results, numerical simulations of the behavior of this chemical system, when periodically forced, resulted in wider tongues for ratios larger than 1.0 (i.e., when the eigenfrequency of the forced oscillator exceeded the forcing frequency).

Experiment 2

The results of Experiment 1 provided a first impression of the regime diagram underlying multifrequency performance in skilled drummers. However, while moving through the

[4] Unfortunately, this suggested explanation for the individual differences cannot be corroborated on the basis of the present results. Although the relation between movement frequency and coupling strength enables within-subject comparisons of different levels of coupling strength, it does not allow for between-subjects comparisons.

Table 2
The 96 Frequency Ratios Tested in Experiment 2

Forced period (ms)	Forcing period (ms) 300	400	500	600	700	800	900	1,000
300	1:1	4:3	5:3	2:1	7:3	8:3	3:1	10:3
400	3:4	1:1	5:4	3:2	7:4	2:1	9:4	5:2
500	3:5	4:5	1:1	6:5	7:5	8:5	9:5	2:1
600	1:2	2:3	5:6	1:1	7:6	4:3	3:2	5:3
700	3:7	4:7	5:7	6:7	1:1	8:7	9:7	10:7
800	3:8	1:2	5:8	3:4	7:8	1:1	9:8	5:4
900	1:3	4:9	5:9	2:3	7:9	8:9	1:1	10:9
1,000	3:10	2:5	1:2	3:5	7:10	4:5	9:10	1:1
1,100	3:11	4:11	5:11	6:11	7:11	8:11	9:11	10:11
1,200	1:4	1:3	5:12	1:2	7:12	2:3	3:4	5:6
1,300	3:13	4:13	5:13	6:13	7:13	8:13	9:13	10:13
1,400	3:14	2:7	5:14	3:7	1:2	4:7	9:14	5:7

Note. Each trial involved a combination of forcing and forced periods.

diagram, the system had to tune into the new frequency ratio in a relatively brief period of time. Just as tuning a radio receiver may be difficult when the knob is turned too quickly, it may have been difficult for the movement system to become attuned to the intended ratio (see Beek & Beek, 1991). In Experiment 2, the participants were, for this reason, instructed to perform several frequency ratios in a stationary fashion.

Method

Participants

The same 3 drummers participated as in Experiment 1. They were paid for their services.

Experimental Setup

The same apparatus was used as in Experiment 1. During each trial, two stimulus trains were presented at different frequencies. The range of interbeep periods presented through the right channel of the headphone was again 300 to 1,000 ms (100-ms steps, 8 different periods). The visual stimulus was presented with the green LED situated close to the tap position of the left hand. The range of frequencies presented through the LED was 300 to 1,400 ms (100-ms steps, 12 different periods). All combinations of the stimulus trains were presented in the experiment (see Table 2). Each trial lasted 30 s.

Procedure

As in Experiment 1, the participant tapped his hands on the marble tabletop while his lower arms rested on the tabletop surface. The instruction was to synchronize the left-hand (forced hand) taps to the visual stimulus and the right-hand (forcing hand) taps to the auditory stimulus. The participant was informed that the prescribed frequencies would not change during a trial. He was instructed to pay close attention to the right (preferred) hand and to make sure that hand tapped the correct frequency, even if this would hamper the performance of the left hand.

The trials were grouped into eight forcing frequency blocks. Before presentation of such a block, the participant was acquainted with the frequency to be tapped by the right hand by means of a unimanual practice trial. During a forcing frequency block, all combinations with the visual frequencies were presented, blocked in series of four trials; the first trial was regarded as a practice trial. These combination blocks were presented in random order. The eight forcing frequency blocks were also presented in random order, divided over three sessions (consisting of either two or three blocks) run on 3 different days. Before the first session, the task was introduced to the participant by means of three randomly selected blocks of four trials. Each forcing frequency block lasted about 50 min. There was a 15-min break between the blocks.

Analysis

The data were analyzed in the same way as in Experiment 1. Because the trials consisted of a single frequency ratio platform, they were all treated in the same manner. The first 8 s and the last second were omitted from analysis. The percentage of the 288 trials that did not pass the stability criterion differed across the participants (Participant A, 12%; Participant B, 10%; and Participant C, 44%).

Results and Discussion

The mean RAEs (averaged over participants and conditions) were 1.5% for the right hand (SD = 1.8%) and 6.3% for the left hand (SD = 4.2%). The corresponding mean CVs were 9.2% (SD = 2.9%) and 13.5% (SD = 4.9%). As a means of examining whether the forcing (right) hand performed more accurately than the forced (left) hand (i.e., conformed to the task instructions), Wilcoxon signed-rank tests for matched pairs (right and left hand) were conducted, separately for each participant, on the RAEs and CVs obtained for the eight forcing frequency conditions. For all 3 participants, the results revealed that the forcing (right) hand performed more accurately: CV, $N = 8$, $T \leq 5$, $p < .05$ (one-tailed), and RAE, $N = 8$, $T \leq 3$, $p < .01$ (one-tailed).[5]

[5] When two-tailed tests were used, all effects but one remained significant: CV, $N = 8$, $T = 4$, $p < .05$ (for Participants B and C), and RAE, $N = 8$, $T \leq 3$, $p < .02$ (for all participants).

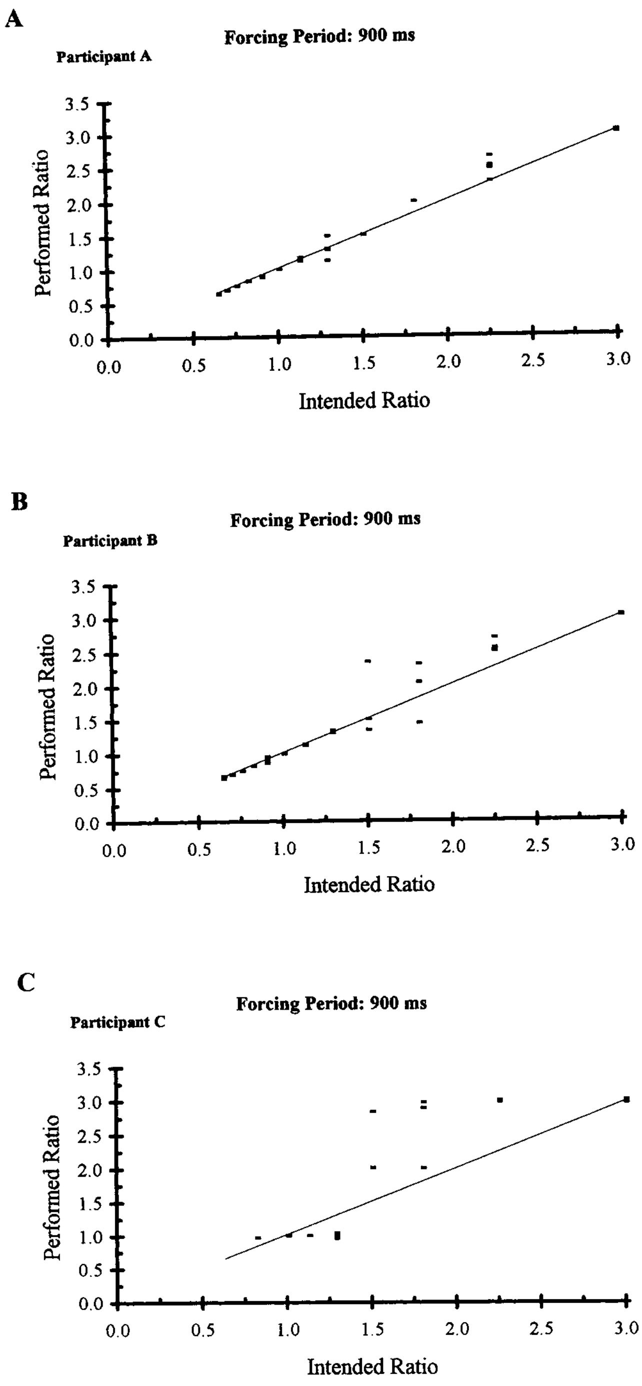

Figure 9. Stable frequency ratios between the forcing hand and the forced hand obtained for a forcing period of 900 ms (for each participant as a function of intended ratio). Thin lines denote locations of correct responses.

As in Experiment 1, the figures that presented the stable frequency locks resembled the devil's staircase and revealed marked individual differences (see Figure 9). Participant A was able to perform a large portion of the intended ratios. For Participant B, the deviations from the intended ratios turned out to be larger, especially for ratios larger than 1.0. Participant C was unable to perform the higher order ratios, showing either unstable behavior or large regions of attraction for the lower order ratios. For all 3 participants, the attraction regions turned out to be the largest for ratios larger than 1.0. Competition between attracting ratios (see Figures 9B and 9C) resulted in a bistable situation similar to that revealed by the hysteresis effects in Experiment 1. These results again suggest that the system was situated above criticality (i.e., the region where Arnold tongues overlap). Inspection of the results for required ratios larger than 1.0 revealed that, for Participant C, the lower order regions of attraction shrank at higher movement frequencies. However, similar to the results obtained in Experiment 1, the previously overlapped higher order ratios did not provide any stability, suggesting that, for this participant, these higher order tongues were not sufficiently developed.

Participants A and B performed the ratios smaller than 1.0 (when the forcing frequency was higher than the frequency of the forced oscillator) quite accurately. For the lower movement frequencies, the performance of Participant C did not show much attraction to lower order ratios; in the high-frequency conditions, however, the system often locked into lower order tongues such as 1:1, 1:2, and 1:3.

To analyze performance in greater detail than is possible from visual inspections of the plots of the performed versus the intended ratios, we also examined the results for each experimental trial (three per condition) numerically. We categorized the performance in each trial along the following lines. The performed ratio was considered to be correct if it was performed stably (CV averaged over the hands

Table 3
Corrected Numbers of Observations of the Three Types of Behavior: Experiment 2

Participant	Ratio ≤1.0	Ratio >1.0	$\chi^2(df = 1)$	*N*	*p*
	Correct lock[a]				
A	74.1	30	18.68	104.1	<.001
B	72.9	47	5.59	119.9	<.02
C	37.1	4	26.66	41.1	<.001
M	61.4	27			
	Other lock[b]				
A	4.1	40	29.22	34.1	<.001
B	1.2	26	22.61	27.1	<.001
C	11.1	41	17.16	52.1	<.001
M	5.5	32.3			
	No lock[c]				
A	5.8	14	3.40	19.8	*ns*
B	9.9	11	0.06	20.9	*ns*
C	35.8	39	0.14	74.8	*ns*
M	17.2	21.3			

Note. The forcing hand moved either faster (≤1.0) or slower (>1.0) than the forced hand.
[a] Stable performance of intended ratio. [b] Attraction to another ratio. [c] No stable performance.

Table 4
Numbers and Types of Observed Transitions Leading to Stable Attraction in Experiment 2, Along With Farey Levels of the Attracting Ratios

Farey level	Farey mod1 down	Farey mod1 up	Farey down	Farey up	Non-Farey	Total
−1	22		7			29
0	22		4			26
1	27	2	12		1	42
2	13	2	12		4	31
3	3		4		4	11
>3	2	1		2	3	8
Total	89	5	39	2	12	147

smaller than or equal to 15%; see Experiment 1) and if, in addition, the mean frequency ratio did not differ by more than ±5% from the intended ratio. If a stably performed ratio fell outside the tolerance range of the intended ratio, the attracting ratio was determined by dissecting the unit interval into Farey F_{12} intervals (i.e., Farey order 12; denominator ≤ 12). If $P_1{:}Q_1$, $P{:}Q$, and $P_2{:}Q_2$ are three adjacent fractions in F_n, the Farey interval assigned to $P{:}Q$ is determined on the basis of Farey's theorem:

$$I_{P,Q} = \left(\frac{P + P_1}{Q + Q_1}, \frac{P + P_2}{Q + Q_2}\right). \tag{4}$$

All rational space was incorporated in the resulting intervals. They did not overlap, and their widths were related to their order: Lower order ratios were assigned larger intervals than higher order ones. The frequency ratio that was performed by the participant was classified by means of the intervals thus obtained. If the ratio was larger than 1.0, the interval was determined by the inverse values of the boundaries obtained according to Equation 4. In this way, the number of trials leading to (a) the intended frequency lock, (b) another frequency lock, or (c) unstable behavior could be determined.

To test the difference between the performance of ratios smaller than or equal to 1.0 and those larger than 1.0, we conducted three chi-square tests for each participant separately: one on the number of correct (intended) locks, one on the number of other (unintended) locks (to which performance was attracted), and one on the number of trials leading to unstable behavior (no locking). Because the number of occurrences of intended ratios up to 1.0 (68 conditions) exceeded the number of ratios larger than 1.0 (28 conditions), the numbers were corrected before analysis: The numbers of observations obtained for the ratios up to 1.0 were multiplied by 7/17 (i.e., 28/68). The results were similar for all 3 participants (see Table 3). Significantly more correct performances occurred for the ratios smaller than or equal to 1.0. The number of trials in which the performance was attracted to another ratio was significantly larger for the ratios larger than 1.0. The number of occurrences of unstable (unlocked) behavior appeared not to differ over the two parts of the regime diagram.

If the intended ratio could not be performed in a stable fashion, the system was often attracted to another ratio. Because detailed individual-specific models were lacking, these transitions were expected most generally to follow the branches of the Farey tree to a lower order ratio. The obtained transition routes were analyzed to examine this prediction. If the attracting ratio was reached via a monotonic route (up or down) following the branches of the tree (representing the dependence on the principle of Farey summation), the transition was referred to as *Farey*. If the transition also satisfied the unimodularity principle (attraction to either a parent or a mediant), it was referred to as a Farey mod1 transition. To investigate the relations between the intended and the performed ratios, we categorized the observed transitions into five classes: Farey mod1 down (to a lower order ratio [a parent]), Farey mod1 up (to a higher order ratio [a mediant]), Farey down, Farey up, and non-Farey (no dependence on the basis of the Farey sum if started with the ratios 0:1 and 1:1). The levels of the attracting ratios (the ratios to which the system was attracted when the intended ratio did not provide sufficient stability) are represented in Table 4, which reports the number of occurrences of each transition type. Clearly, most transitions followed a Farey mod1 down transition route, especially when the attracting ratio was situated at a low level in the tree. By far the largest part of the remaining transitions also followed the branches of the Farey tree but did not directly end up at a unimodularly related ratio. Inspection of Table 4 reveals that these transition routes were situated a little higher in the tree. A Farey down transition implies that the intended ratio was at least two levels higher in the tree (e.g., a Farey down transition to Level 1 implies that the intended ratio was at least situated at Level 3). Because, in general, ratios at the higher levels are less stable, it is not surprising that many unimodularly related ratios did not provide stability either. The latter were mostly situated fairly high in the tree as well, so that, as a result of fluctuations in behavior (noise), the resulting transitions tended to jump over these ratios. Because the Farey tree reflects the spatial organization of the Arnold tongues in the regime diagram, the fact that by far the most transitions followed the structure of the Farey tree (to a lower level) revealed that performance at unstable regions was, in general, attracted to a close lower order ratio.

Table 5
The 15 Frequency Ratios Tested in Part 1 of Experiment 3

Forced period (ms)	Forcing period (ms)		
	500	700	900
400	5:4	7:4	9:4
600	5:6	7:6	3:2
800	5:8	7:8	9:8
1,000	1:2	7:10	9:10
1,200	5:12	7:12	3:4

Note. Each trial involved a combination of forcing and forced periods.

Table 6
Corrected Numbers of Incorrect Locks in Part 1 of Experiment 3

	Ratio		
Condition	≤1.0	>1.0	Total
RA	0.67	9	9.67
RV	4.67	4	8.67
LA	2.00	8	10.00
LV	5.33	4	9.33
Total	12.67	25	37.67

Note. The forcing hand moved either faster (≤1.0) or slower (>1.0) than the forced hand. The first letter of each row label refers to the hand performing the role of forcer (R = right; L = left), and the second letter refers to the stimulus modality to which the hand synchronized (A = auditory; V = visual).

In sum, the present results are similar to those of Experiment 1. If the forcing frequency was higher than the frequency of the forced hand, Participants A and B were well able to perform the intended ratios, particularly if the movement frequency was low. In this domain of the regime diagram, attraction to other ratios was rarely observed for any of the participants. However, if the forced oscillator moved faster (i.e., if the ratio was larger than 1.0), higher order ratios appeared to be far less stable, the performance being significantly more often attracted to ratios at a lower level in the Farey tree. The observed transitions followed the branches of the Farey tree in the vast majority of cases (predominantly according to the unimodularity principle), thereby supporting the assumption that unstable performance may be attracted to a nearby lower order ratio.

The larger difference observed between performance of ratios smaller than 1.0 and those larger than 1.0 might have been caused by a number of factors. In both Experiments 1 and 2, the right (preferred) hand functioned as the forcing oscillator, following an auditory stimulus. The observed difference may therefore be due to (a) the difference in role (forcing vs. forced oscillator) assigned to the hands by means of instruction, (b) differences between the hands (right vs. left), or (c) the difference between the stimuli (auditory vs. visual). A control experiment was conducted to examine these possibilities.

Experiment 3

Experiment 3 consisted of two parts: A unimanual and a bimanual study examined the influences of the effects of hand (left vs. right) and stimulus (auditory vs. visual). In the bimanual study, the additional factor of role (forcing vs. forced oscillator) was tested. In this experiment, the role of forcer was determined by instruction only, whereas the conditions under which the hand performed varied.

Method

Participants

The same drummers participated as in Experiments 1 and 2. They were paid for their participation.

Experimental Setup

The same setup was used as in the previous experiments. An extra LED (visual stimulus) was positioned near the tapping position of the right hand.

Part 1. A subset of the frequency combinations used in Experiment 2 was tested in this experiment. The interstimulus period for the forcing hand was 500, 700, or 900 ms. The interstimulus period for the other hand was 400, 600, 800, 1,000, or 1,200 ms. Each stimulus train could be presented by means of either an auditory or a visual stimulus for either the left or the right hand (using the corresponding LED or headphone channel). The resulting frequency ratios are presented in Table 5.

Part 2. Only one stimulus train was generated. It could be presented either visually or acoustically for either the left or the right hand. The six interstimulus periods tested were 400, 600, 800, 1,000, 1,200, and 1,400 ms.

Procedure

The instruction for Part 1 was similar to that of Experiment 2. One of the hands was assigned the role of forcer by means of the instruction to pay close attention to that hand and to perform the

Table 7
Results of Two Nonparametric Analyses of Variance Performed for the Individual Participants on the Absolute Error and on the Coefficient of Variation: Part 1 of Experiment 3

	Role		Hand		Stimulus		Role × Hand		Role × Stimulus		Hand × Stimulus		Role × Hand × Stimulus	
Participant	χ^2	p	χ^2	p	χ^2	p	χ^2	p	χ^2	p	χ^2	p	χ^2	p
							Absolute error							
A	—	—	—	—	—	—	—	—	—	—	4.8	<.05	—	—
B	3.3	<.10	—	—	—	—	5.1	<.05	19.2	<.001	3.2	<.10	—	—
C	3.3	<.10	—	—	10.8	<.01	—	—	13.3	<.001	6.5	<.05	—	—
							Coefficient of variation							
A	—	—	—	—	—	—	—	—	—	—	8.5	<.01	—	—
B	—	—	—	—	—	—	—	—	35.1	<.001	—	—	—	—
C	—	—	—	—	—	—	4.5	<.05	—	—	—	—	—	—

Note. Effects at or below the .10 probability level are reported (see text for details). Dashes indicate that effects were above the .10 probability level. For all χ^2 values, N = 120 and df = 1.

intended period for it correctly, even if this would hamper the performance of the other hand. This part of the experiment consisted of four subdivisions: The forcing hand was either left or right, and the stimulus pacing this hand was either auditory or visual. The role of forced oscillator was performed by the other hand synchronizing to the stimulus of the other modality. The order in which these subdivisions were tested was semicounterbalanced over the participants. For each subdivision, all combinations of forcing and forced frequencies were presented. The forcing frequencies were presented grouped together, resulting in three forcing frequency blocks per subdivision. The combinations with the other stimulus train were blocked together (four trials per block, the first of which served as a practice trial). These blocks were presented in a random order. Before each forcing frequency block, a unimanual practice trial was conducted to familiarize the participant with the required forcing frequency. Before the experimental trials of a subdivision were started, the participant was acquainted to the modality–hand combination by means of a set of practice trials. For a randomly chosen forcing frequency, the participant received, after the accompanying unimanual practice trial, two randomly selected subblocks of two trials. Part 1 was conducted on 2 different days (two subdivisions per day). On the 2nd day, Part 2 of the experiment was conducted as well. In this part of the experiment, the participant tapped unimanually with either hand paced by either a visual or an auditory stimulus. The combinations of performing hand and modality were tested groupwise (semicounterbalanced across participants). The six tapping frequencies were presented in blocks of three trials in random order. In both parts of the experiment, the trials lasted 30 s each.

Analysis

The tapping periods were determined in the same way as in the previous experiments. For both parts of the experiment, the mean intertap period and the CV were determined for each trial (excluding the first 8 s and the last second from analysis). For Part 1 (bimanual performance), we obtained the mean frequency ratio by dividing the mean intertap period of the forcing hand by that of the forced hand. Stable performance was determined in the same way as in the previous experiments.

Results

Part 1: Bimanual Performance

The number of incorrect locks (when behavior was attracted to ratios other than the intended one) are presented in Table 6. (Because the number of intended ratios smaller than 1.0 [9] was larger than the number of such ratios larger than 1.0 [6], the numbers of observations were corrected by multiplying the number of observations in the first category by two thirds.) The effects of condition (forcing hand right paced acoustically vs. forcing hand right paced visually vs. forcing hand left paced acoustically vs. forcing hand left paced visually) and ratio type (smaller than 1.0 vs. larger than 1.0) were examined in a chi-square analysis under the null hypothesis that no significant differences existed among the eight situations. The analysis revealed the opposite result, $\chi^2(3, N = 37.67) = 11.59, p < .01$. Table 6 *indicates that this effect* was probably due to the differences between the ratio types, the stimulus modalities (as presented for the forcing hand), or the combination of these factors. Additional chi-square tests (under the same null hypothesis) showed that both factors significantly influenced the behavior. A one-sample chi-square test on the effect of ratio type revealed that significantly more transitions to other ratios occurred when the intended ratio was larger than 1.0, $\chi^2(1, N = 37.67) = 4.04, p < .05$. The main effect of stimulus modality was investigated with a similar test. This effect was not significant, $\chi^2(1, N = 37.67) = 0.07, p > .70$. A 2 (ratio type) × 2 (stimulus) chi-square test, however, revealed a significant interaction effect, $\chi^2(1, N = 37.67) = 11.09, p < .001$. This occurred because the effect of ratio type was present in the auditory conditions but vanished when the stimulus pacing the forcing hand was visual.

The implications of this result cannot readily be understood. In the auditory condition, the number of trials in which performance was attracted to an unintended frequency lock was significantly larger if the forcing hand (synchronizing to the auditory stimulus) tapped at a lower frequency than the forced hand. If the difference in performance for the two ratio types was simply a stimulus effect, the reverse effect would be observed in the visual condition (forcing hand synchronizing to the visual stimulus); the number of unintended locks would then be expected to be lower in case the forcing hand was tapping slower than the forced hand. However, such a reversal was not observed. In the visual condition, the difference between performance of ratios smaller than 1.0 and those larger than 1.0 simply disappeared. To gain more insight into this asymmetrical stimulus effect, we carried out additional analyses of the effects of the experimental conditions on the performance of the component oscillators.

Because the tested ratios differed over forcing frequency conditions and performance thus was influenced by different attraction regions in each condition, the results were interpreted at the ordinal scale. Therefore, the obtained intertap periods and CVs (averaged over the three experimental trials that were conducted for identical conditions) were examined with the nonparametric analysis of variance (ANOVA) developed by Wilson (1956). First, two 2 × 2 × 2 ANOVAs with the variables of role (forcing vs. forced), hand (left vs. right), and stimulus (auditory vs. visual) were conducted. One concerned the absolute deviations of the mean intertap periods from the intended periods (absolute error), and the other tested the effects on the CVs. The ANOVAs were performed for each of the 3 participants separately. The results are presented in Table 7. With respect to the absolute errors, two main effects were observed for Participant C. First, if a hand was to synchronize to the auditory stimuli, its intertap period was significantly closer to the intended period than when synchronizing to the visual stimuli. Second, if a hand was assigned the forcing role, performance tended to be significantly better than when it functioned as the forced oscillator. The two interaction effects revealed that the auditory stimulus, in combination with either the forcing hand or the right hand, led to superior performance in comparison with other role–stimulus and hand–stimulus combinations, respectively. The main effect of role and the Role × Stimulus interaction obtained for Participant B revealed that, for this participant as well, the forcing hand performed more accurately than the forced hand, especially when it was paced by the auditory stimulus. The interaction effect between hand and stimulus observed for this participant revealed that if the left hand synchronized to the visual stimulus, performance was worse than in the other three role–stimulus conditions. In addition, the Role × Hand interaction showed that assigning the role of forcer to the left hand resulted in more accurate performance than that obtained in the other three combinations of role and hand. Participant A performed significantly better with the right hand paced by the auditory stimulus than in the other hand–stimulus conditions. The ANOVAs on the CVs revealed no main effects. The interaction effects obtained for the individual participants showed that Participant A performed less variably with the right hand if it synchronized to the auditory stimulus (relative to the other hand–stimulus conditions). The performance of Participant C was by far the most variable if the right hand assumed the role of forced oscillator as opposed to the other combinations of role and hand. For Participant B, the forcing hand performed significantly better when synchronizing to the auditory stimulus as opposed to the visual stimulus; for the forced hand, the reverse was true. Because auditory pacing of the forcing hand was always accompanied by visual pacing of the forced hand, these results reveal that, for Participant B, this experimental condition led to superior performance.

In this experiment, we attempted to ensure the forcing role performed by one of the hands by instructing the participant to perform correctly with this hand even at the expense of the other hand's performance. Because of the crucial role of the forcing hand, we conducted additional tests to examine the effects of the experimental conditions on the performance with this hand. We performed 2 (hand) × 2 (stimulus) nonparametric ANOVAs on both the absolute errors and the CVs, and again we conducted the tests for the individual participants. For each participant, the main effect of stimulus on the absolute errors was significant, revealing that performance of the forcing hand was best when tapping was synchronized to the auditory stimulus: Participant A, $\chi^2(1, N = 60) = 4.27, p < .05$; Participant B, $\chi^2(1, N = 60) = 4.26, p < .05$; and Participant C, $\chi^2(1, N = 60) = 17.07, p < .001$. In addition, a significant interaction effect in favor of the right hand–auditory stimulus condition (in comparison with the other hand–stimulus conditions) was demonstrated for Participant C, $\chi^2(1, N = 60) = 4.47, p < .05$. For the CVs, only two significant effects were found. Participant B performed significantly better with the forcing hand if it synchronized to the auditory stimulus as opposed to the visual stimulus, $\chi^2(1, N = 60) = 17.10, p < .001$. For Participant A, the condition in which the right hand paced by the auditory stimulus functioned as forcing oscillator resulted in the best performance, $\chi^2(1, N = 60) = 6.67, p < .01$.

Part 2: Unimanual Performance

On average, the absolute timing error relative to the intended period (RAE) was only 0.49% (*SD* = 0.96%). A 2 (hand: left vs. right) × 2 (stimulus: visual vs. auditory) × 6 (movement frequency: six levels) ANOVA with repeated measures on the RAEs revealed no significant effects of the tested variables. A similar ANOVA was performed on the CVs obtained for the performed intertap periods. Again, no significant effects were observed (mean CV = 4.39%, *SD* = 1.98%).

Discussion

The unimanual part of this experiment revealed no significant differences either between the hands (left or right) or between the modalities of the pacing stimulus (auditory or visual). In bimanual performance of the presented frequency ratios, however, the timing accuracy of the taps was affected by the experimental conditions. Although the effects differed over the participants, some general tendencies can be deduced from these results. In general, the performance of the acoustically paced hand was more accurate than that of the visually paced hand. The performance of the right hand was, in a number of cases, superior to that of the left hand, and the forcing hand performed better than the forced hand (the latter being in agreement with the task instructions). In the analysis of the performance of the forcing hand, the benefit of the auditory stimulus became even clearer. Thus, the forcing role of a hand is best guaranteed if it is the right hand paced by an auditory stimulus. In fact, this confirmed our earlier expectation based on influences of handedness and stimulus modality as reported in the literature, which motivated the designs of Experiments 1 and 2.

The results of Experiment 3 indicated that the difference in the number of transitions that occurred for the ratios smaller than 1.0 (i.e., forcing hand tapping with a higher frequency than the forced hand) and for those larger than 1.0 (i.e., forcing hand tapping the lower frequency), as observed in Experiments 1 and 2, was not simply an artifact of differences between the left and right hand or between the modalities of the stimulus pacing a hand. If such were the case, changing the forcing role from the right to the left hand or changing its pacing stimulus from auditory to visual would have reversed the difference in the number of unintended locks for the two ratio types. However, this is not what happened. The difference in the number of transitions to unintended ratios was not significantly affected by changing the forcing and forced role between the hands. Moreover, when the forcing hand synchronized to the visual stimulus instead of the auditory stimulus, the difference in the number of transitions for ratios smaller than 1.0 and those larger than 1.0 did not reverse but vanished. Given that no main effect of the stimulus variable was obtained, this means that the number of transitions observed for ratios smaller than 1.0 increased, whereas the number of transitions obtained for ratios larger than 1.0 decreased (see also

Table 6). Generally speaking, performance was less accurate when taps were paced by a visual stimulus than when they were paced by an auditory stimulus. These findings strongly suggest that the instruction per se (i.e., to attend closely to one of the hands and to perform most accurately with it) is not sufficient to ensure that this hand will indeed adopt the forcing role. In particular, this task requirement proved difficult to meet if the hand in question had to synchronize to the visual stimulus. In the latter condition, the role of forcer seemed to be ill defined.

Therefore, one may conclude that, in those situations in which the participants were able to attain the role of forcer with the designated hand (i.e., when it was paced by the auditory stimulus), the increase in the number of transitions observed if the forcing hand moved slower than the forced hand was not an artifact of the experimental conditions. Rather, it was due to the relation between the forcing and the forced frequency per se.

General Discussion

Three experiments were conducted to examine multifrequency behavior in skilled tapping. A number of interesting dynamical phenomena were observed, including entrainment, bistability, and hysteresis. In addition, it was demonstrated that the stability of behavior was affected by movement frequency. These phenomena are not captured by current timekeeper models (e.g., Jagacinski et al., 1988; Summers, Rosenbaum, et al., 1993) because these models are not suited to addressing stability. Because differential stability and changes therein (crucial features of our results) form a (if not the) key problem of movement coordination, dynamical modeling offers a more fruitful and parsimonous way to understand the observed behavior than do representational approaches.

Nonlinear dynamics provides tools to model the rhythmically tapping hands in an abstract way, regardless of the structural properties of the subsystems and the exact mechanism of their interaction. Collectively, our results supported the notion that, in performing bimanual multifrequency tasks, the movement system may be modeled as a system of nonlinearly coupled nonlinear oscillators. The discrete circle map provides a relatively simple and general window into the behavior of such systems. In qualitative agreement with the circle map, regions of attraction were found when the system was slowly scaled through a range of intended frequency ratios. In general, lower order ratios were associated with larger regions of attraction than higher order ratios. In the vast majority of cases, the relations between the intended ratios and the ratios to which the system was attracted were in agreement with the branching structure of the Farey tree. In general, the performance was attracted to a nearby lower order ratio. The larger number of Farey mod1 transitions disclosed that most transitions were actually to one of the two closest lower order ratios (parents). This is exactly what would be expected on the basis of the adopted theoretical framework.

Transition Mechanisms

By manipulating the movement frequency (demonstrated to be inversely related to coupling strength by Schmidt et al., 1993, and Sternad et al., 1992) at which the intended ratios were to be performed, we were able to study the behavior for different degrees of interaction between the limbs. As expected, the stability of performance decreased as the ratios were performed at higher frequencies. Two mechanisms leading to the observed transitions could be inferred from the data, both resulting from deterministic as well as stochastic properties of the movement system. The first type of transitions were caused by the lack of stability of the higher order ratios at high movement frequencies, in the sense that the tongues were too narrow to resist stochastic fluctuations. As a result, the system was kicked out of the resonance region and trapped by an attractor corresponding to a lower order ratio. At low frequencies, however, particularly for ratios larger than 1.0, attraction regions were observed that were too large and too asymmetrical for this kind of mechanism to be plausible. The attractions and, moreover, the bistability and hysteresis effects observed for 2 of the 3 participants suggested that, at least in some cases, the Arnold tongue for the intended ratio was overlapped by another resonance zone so that stochastic fluctuations resulted in a transition to this overlapping tongue. According to our model, the one-dimensional circle map, the occurrence of this second transition mechanism implied that the system was situated in the supercritical domain of the map where periodic solutions may overlap. Recently, this interpretation was deemed unlikely by Treffner and Turvey (1993) on the grounds that above the critical line, mode locking is unavoidable and leads to deterministic chaos. However, although it is true that chaos results from overlapping Arnold tongues and is therefore exclusively present in the supercritical domain, other forms of behavior are not excluded here: The overlap between resonance zones may also lead to bistability and hysteresis, whereas in the nonoverlapping parts of the tongues the behavior remains periodic. Thus, only above the critical line does one find the richness of behavior observed in Experiments 1 and 2. Although the present results (as well as those of Treffner & Turvey, 1993) are too premature to allow for any definitive conclusions about where exactly in the regime diagram the movement system is predominantly situated, the results of Experiments 1 and 2 suggest that, at least under some conditions, it is situated in the supercritical domain. This would imply that transitions may occur not only because resonance zones are too narrow to resist stochastic perturbations but also because there is overlap between the tongues.

Is "Unstable" Behavior Chaotic?

In the subcritical domain of the circle map, the ever-present variations in (average) periodic performance can be interpreted as quasiperiodic performance close to the boundaries of the Arnold tongues with the aim to preserve both stability and flexibility (see Beek, 1989a). In the supercritical domain, however, quasiperiodic behavior cannot occur, and the unstability in performance must therefore be interpreted as either chaotic or multiperiodic. Differentiating between these two possibilities is troublesome. In principle, it boils down to discriminating stochastic from deterministic noise. If the behavior is periodic, variability results from stochastic noise. Chaotic behavior in the nonlinear dynamical sense implies that the system is highly sensitive to changes in initial conditions. This sensitivity is expressed in time series by rapid divergence of initially nearby points in phase space. Although chaotic behavior appears to be highly irregular, it is deterministic and shows a particular kind of regularity; certain regions in phase space appear to be inaccessible, and a so-called "strange attractor" with a fractal dimension underlies the behavior. The presence of such an attractor may be revealed by using dimensionality analyses (e.g., correlation or Lyaponov dimensions; e.g., see Farmer, Ott, & Yorke, 1983). However, these types of analyses are complicated and require long time series and many cycles. Experimental runs in human movement research are typically too short to satisfy these criteria. A further complicating factor is that, ideally, the dynamics of the system in these time series should be stationary (i.e., the attractor layout should not change), a requirement that need not be met in experimentally obtained time series. These may be two of the reasons why, until now, chaos has not been demonstrated in human movement.

Asymmetrical Coupling

In the conditions in which the forcing hand followed the auditory stimulus, a marked difference between the situations in which the forcing hand moved either faster or slower than the forced oscillator was obtained. If the visual stimulus paced the forcing hand, this difference disappeared. Detailed analyses revealed that this was probably the result of difficulties in meeting the task requirements. If the task was performed correctly (i.e., in the conditions in which the forcing hand synchronized to the auditory stimulus), the ratios up to 1.0 (forcing hand faster than forced hand) were performed far more accurately than those larger than 1.0 (forcing hand slower than forced hand). In the latter situation, the system was often attracted to one of just a few lower order ratios. The results resemble those of numerical studies of the Brusselator model (M. Dolník, personal communication, November 18, 1993, and September 12, 1994; Kai & Tomita, 1979; Schreiber et al., 1988). The simulations of the behavior of this model chemical oscillator, when periodically forced, showed that the regions of attraction were much larger if the forcing frequency was lower than the frequency of the forced chemical system. In the simulated system, the coupling between the two oscillators was unidirectional: The external periodic stimulus forcing the chemical oscillator was not influenced by this forced oscillator. In our system we attempted, through conditions and task instructions, to come close to such a situation. Although in bimanual performance the hands are influencing each other (e.g., see the HKB model; Haken et al., 1985; Schmidt, Beek, Treffner, & Turvey, 1991), the similarity between the behavior of the unidirectionally forced Brusselator model and our results may indicate that the coupling between the hands was indeed asymmetrical, resulting in a larger influence of the forcer on the forced oscillator than vice versa.

In the visual conditions (forcing hand synchronizing to the visual stimulus), however, no differences were observed between the situations in which the forcing hand moved either faster or slower than the forced hand. Analyses of the accuracy of the forcing hand suggested that the role of forcer was ill defined if this hand was paced by the visual stimulus. Following the earlier line of thought, this may have resulted in a more symmetrical coupling between the hands and, consequently, in a loss of distinction between the forcing and the forced oscillator. In that case, the categories "larger than 1.0" and "smaller than 1.0" lose their meaning, and no differences between these ratio types are to be expected.

Interestingly, a number of studies on polyrhythmic performance have demonstrated that the fast hand generally performs more accurately than the slow hand. Our study confirms the suggestion that this is not a handedness effect (Summers, Ford, & Todd, 1993; Summers, Rosenbaum, et al., 1993). In addition, Peters and Schwartz (1989) demonstrated that performance of the 2:3 polyrhythm is more accurate if attention is focused (by means of counting aloud along with the taps) on the hand (either left or right) tapping the faster frequency. These effects were interpreted as evidence that the coordination of such rhythmic movements is governed by a hierarchical control scheme based on a single timing mechanism assumed to be related to the fast frequency. Moreover, observed differences in coordination between musicians and nonmusicians suggested that whereas in musicians the timing of the slow hand was made dependent on the timing of the fast hand, nonmusicians were not able to integrate the taps of the slow hand into the timing structure of the fast hand (Summers, Ford, & Todd, 1993; Summers & Kennedy, 1992). In general, the performance of musicians is reported to be superior to that of nonmusicians, indicating that the integrated interleaving of the slow taps is the skilled way to solve the problem of bimanually performing two different frequencies.

Our results, however, suggest an alternative explanation of these general findings. Because the performance obtained for the ratios smaller than 1.0 (forcing hand taps faster than forced hand) in the auditory condition resulted in fewer transitions to unintended ratios than that obtained for the visual conditions, in which the role of forcer was ill defined (see Table 6), one may conclude that a situation in which the fast hand functions as a forcer is beneficial. In other words, an asymmetrical coupling seems to allow for more accurate performance than symmetrical coupling, as long as the forcing hand moves the fastest (regardless of whether it is the left or the right hand). With respect to the differences between skilled and unskilled participants, this suggests that extended practice results in an asymmetrical coupling between the limbs.

Although Experiment 3 did not reveal any differences in the accuracy of unimanual performance, as obtained when synchronizing to either an auditory or a visual stimulus, synchronization of the forcing hand to the visual stimulus in the bimanual conditions led to problems in meeting the task requirements, whereas synchronizing it to the auditory stimulus did not. Attending to an auditory stimulus may be less demanding than attending to a visual stimulus, and this difference may be reflected in more stable behavior of acoustically paced limbs than visually paced limbs when attentional demands are high (as in our bimanual task). At any rate, the influence of stimulus modality underscores the point that coordination dynamics is not just "ordinary" physics. The collective variables governing behavior are context-sensitive informational variables, and the laws of coordination are more general than the particular structures that embody them (Kelso, 1994).

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